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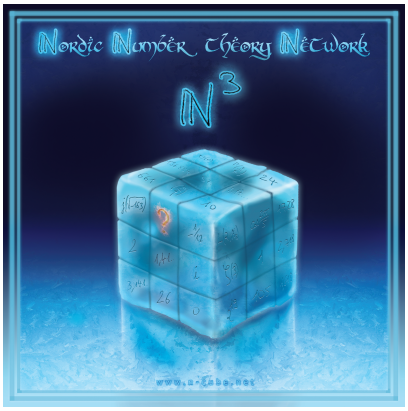
# Nordic Number theory Network Days XVIII

Aarhus University,
July 10 – July 12, 2023
organized by Maiken B. Gravgaard (Aarhus), Simon Kristensen (Aarhus),
and Mathias L. Laursen (Aarhus).




## Program

| | Monday 10.07 | Tuesday 11.07 | Wednesday 12.07 |
|---|---|---|---|
| 10:00–10:50 | | **Tapani Matala-aho** | **Demi Allen** |
| 10:50–11:10 | | *Coffee Break* | *Coffee Break* |
| 11:10–12:00 | | **Radhakrishnan Nair** | **Oleg Karpenkov** |
| 12:00–12:30 | | *Lunch* | *Farewell* |
| 12:30–13:30 | *Registration* | | |
| 13:30–14:20 | **Faustin Adiceam** | **Pär Kurlberg** | |
| 14:20–14:40 | *Coffee Break* | *Coffee Break* | |
| 14:40–15:30 | **Jaroslav Hančl** | **Ulf Kühn** | |
| 15:30–15:50 | *Coffee Break* | *Coffee Break* | |
| 15:50–16:40 | **Shreyasi Datta** | **Sanju Velani** | |
| ... | | ... | |
| 18:30–??:?? | | *Conference Dinner* | |

# Abstracts

TIME: Monday 10, 13:30–14:20.
SPEAKER: **Faustin Adiceam** (University of Manchester).
TITLE: *Some Algebraic Tools in Metric Number Theory.*
ABSTRACT: The talk will present some recent advances at the crossroads between Number Theory and Fractal Geometry requiring the input of algebraic theories to estimate the measure and/or the factal dimension of sets emerging naturally in Diophantine Approximation. Examples include the proof of metric, uniform and quantitative versions of the Oppenheim conjecture generalised to the case of any homogeneous form and also the determination of the Hausdorff dimension of the set of well approximable points lying on polynomially defined manifolds (i.e. on algebraic varieties).

TIME: Monday 10, 14:40–15:30.
SPEAKER: **Jaroslav Hančl** (Universiy of Ostrava).
TITLE: *Expresion of real numbers.*
ABSTRACT: The talk deals with the expression of the real numbers by infinite series and continued fractions. Some recent results and open problems will be presented also.

TIME: Monday 10, 15:50–16:40.
SPEAKER: **Shreyasi Datta** (University of Michigan).
TITLE: *Quantitative simultaneous approximation.*
ABSTRACT: In a recent ground-breaking work (arXiv:2105.13872), Beresnevich and Yang proved Khintchine's theorem in the set-up of simultaneous Diophantine approximation for nondegenerate manifolds, resolving a long-standing problem. In this talk, we will explain an effective version of their result.

This is based on the work https://arxiv.org/abs/2209.14196.

TIME: Tuesday 11, 10:00–10:50.
SPEAKER: **Tapani Matala-aho** (Aalto University).
TITLE: *Euler's factorial series, Hardy integral, and continued fractions.*
ABSTRACT: Let $p$ be a prime and let

$$E_p(t) = \sum_{k=0}^{\infty} k! t^k$$

denote the Euler's factorial series. We will present recent results on lower bounds for the $p$-adic absolute value of the expression $dE_p(p^a) - c$, where $a, c, d \in \mathbb{Z}$. The proofs are based on the fact that the same Padé polynomials which $p$-adically converge to $E_p(t)$, approach the Hardy integral

$$\mathcal{H}(t) = \int_0^{\infty} \frac{e^{-s}}{1 - ts} ds$$

on the Archimedean side. Furthermore, we will discuss on an interconnection between $E(t)$ and $\mathcal{H}(t)$ via continued fractions.

The results are based on joint works with
*Anne-Maria Ernvall-Hytönen, Louna Seppälä and Wadim Zudilin.*

TIME: Tuesday 11, 11:10–12:00.
SPEAKER: **Radhakrishnan Nair** (University of Liverpool).
TITLE: *On Ergodic Theorems and the Riemann Hypothesis.*
ABSTRACT: We use subsequence ergodic theorems applied to Boole's transformation and its variants and their invariant measures on the real line to give new characterisations of the Lindelof Hypothesis and the Riemann hypothesis. This builds on earlier work of R. L. Adler and B. Weiss, M. Lifshits and M. Weber, J. Steuding, J. Lee and A. I. Surijaya using Birkhoff's ergodic theorem and probability theory. The talk is on work with Jean-Louis Verger-Gaugry and Michel Weber.

TIME: Tuesday 11, 13:30–14:20.
SPEAKER: **Pär Kurlberg** (KTH Royal Institute of Technology).
TITLE: *Poisson spacings for lattice points on circles.*
ABSTRACT: We will investigate the distribution of $\mathbb{Z}^2$-lattice points lying on circles. Along a density one subsequence the angles of lattice points on circles are known to be uniformly distributed as the radius tends to infinity; in fact the angles are "very well distributed" in the sense of the discrepancy being *lower* than that of a random collection of points. A refined question is how lattice points are spaced at the local scale, i.e., when rescaled so that the mean spacing is one. I will discuss recent joint work with Steve Lester in which we show that the local spacing statistics are Poissonian along a density one subsequence of admissible radii.

TIME: Tuesday 11, 14:40–15:30.
SPEAKER: **Ulf Kühn** (University of Hamburg).
TITLE: *Recent Developments in Multiple q-zeta Values.*
ABSTRACT: Multiple q-zeta values are a priori $q$-analogs of multiple zeta values, but they turn out to be closely related also to modular forms, multiple Eisenstein series and polynomial functions on partitions. This talk will be a survey on conjectures and recent results for the algebra of multiple q-zeta.

TIME: Tuesday 11, 15:50–16:40.
SPEAKER: **Sanju Velani** (University of York).
TITLE: *The Shrinking Target Problem: Developing a manifold theory.*
ABSTRACT: Let $T$ be a $d \times d$ matrix with integral coefficients. Then $T$ determines a self-map of the $d$-dimensional torus $\mathbb{T}^d = \mathbb{R}^d/\mathbb{Z}^d$. Choose for each natural number $n$ a ball $B(n)$ in $X$ and suppose that $B(n+1)$ has smaller radius than $B(n)$ for all $n$. Thus the ball shrinks as $n$ increases. Now let $W$ be the set of points $x \in \mathbb{T}^d$ such that $T^n(x) \in B(n)$ for infinitely many $n \in \mathbb{N}$. The size of $W$ measured in terms of $d$-dimensional Lebesgue measure (restricted to $\mathbb{T}^d$) and Hausdorff dimension are pretty much well understood. In this talk I explore the situation in which the points $x \in \mathbb{T}^d$ are restricted to a nice subset $\mathcal{M}$ (such as an analytic sub-manifold) of $\mathbb{T}^d$; that is, the points of interest are functionally dependent. I will essentially concentrate on the situation when $d = 2$, $T$ has first row $(2, 0)$ and second row $(0, 3)$ and $\mathcal{M}$ is the diagonal. In this special case, given a decreasing function $\psi$, understanding the shrinking target set $W \cap \mathcal{M}$ is equivalent to understanding the set of $x \in [0, 1]$ such that $\max\{\|2^n x\|, \|3^n x\|\} < \psi(n)$ for infinitely many $n \in N$.

This is joint work with Bing Li (South China University of Technology), Lingmin Liao (UPEC) and Evgeniy Zorin (York).

TIME: Wednesday 12, 10:00–10:50.
SPEAKER: **Demi Allen** (University of Exeter).
TITLE: *Inhomogeneous Diophantine Approximation for systems of linear forms with primitivity constraints.*
ABSTRACT: Diophantine Approximation is a branch of Number Theory in which the central theme is understanding how well real numbers can be approximated by rationals. In the most classical setting, a $\psi$-well-approximable number is one which can be approximated by rationals to a given degree of accuracy specified by an approximating function $\psi$. Khintchine's Theorem provides a beautiful characterisation of the Lebesgue measure of the set of $\psi$-well-approximable numbers and is one of the cornerstone results of Diophantine Approximation.

In this talk I will discuss the generalisation of Khintchine's Theorem to the setting of approximation for systems of linear forms. More specifically, I aim to discuss inhomogeneous Diophantine approximation for systems of linear forms subject to certain primitivity constraints. In one direction, we answer questions posed in this area by Dani, Laurent, and Nogueira (Mathematische Zeitschrift, 2015). In another, slightly different direction, we prove a univariate inhomogeneous version of the Duffin–Schaeffer Conjecture for systems of linear forms in at least three variables. This talk will be based on joint work with Felipe Ramírez (Wesleyan, US).

TIME: Wednesday 12, 11:10–12:00.
SPEAKER: **Oleg Karpenkov** (University of Liverpool).
TITLE: *On Hermite's problem, Jacobi-Perron type algorithms, and Dirichlet groups.*
ABSTRACT: In this talk we introduce a new modification of the Jacobi-Perron algorithm in the three dimensional case. This algorithm is periodic for the case of totally-real conjugate cubic vectors. To the best of our knowledge this is the first Jacobi-Perron type algorithm for which the cubic periodicity is proven. This provides an answer in the totally-real case to the question of algebraic periodicity for cubic irrationalities posed in 1848 by Ch. Hermite.

We will briefly discuss a new approach which is based on geometry of numbers. In addition we point out one important application of Jacobi-Perron type algorithms to the computation of independent elements in the maximal groups of commuting matrices of algebraic irrationalities.