The Two Careers of Emmy Noether
A notable career in Nineteenth Century Erlangen.

The long Nineteenth Century.
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An unfortunate parallel case:
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An unfortunate parallel case:

Mildred Sanderson, 1889–1914.

- Master's and Ph.D. with L.E. Dickson, Chicago.
- Thesis “Formal Modular Invariants with Applications to Binary Modular Covariants.”
Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.
- Deutsche Mathematiker-Vereinigung 1909.
- Supervised Hans Falckenberg dissertation 1911 (with E. Schmidt).
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Then a different career: different place, different life, and a different century in world history and in mathematics.
Life in Erlangen

Weyl: “There was nothing rebellious in her nature; she was willing to accept conditions as they were.”

Weyl probably heard this from her brother Fritz, and it is probably true.
However, she may have irritated Friedrich Nietzsche when she was 5 years old.
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Her lowest grade was “satisfactory,” for practical classroom conduct.
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A student and friend was, Olga Taussky-Todd:
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A student and friend was, Olga Taussky-Todd:
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- “She said women should not try to work as hard as men.
- She remarked that she, on the whole, only helped young men to obtain positions so they could marry and start families.
- She somehow imagined that all women were supported.”
The Turn of the Century in Mathematics.
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I define these two mathematical centuries in relation to foundations.
The Turn of the Century in Mathematics.

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Not (necessarily) formal foundations.
A broad consensus as to:

- What mathematics deals with: numbers?
A broad consensus as to:

- What mathematics deals with: numbers? symbols?

- What do we assume about them at base?

- What questions may legitimately be asked about such entities?

- What counts as a solution?

- Non-constructive proofs?
A broad consensus as to:

▶ What mathematics deals with: numbers? symbols? quantities?
A broad consensus as to:

- What mathematics deals with: numbers? symbols? quantities? sets?

Non-constructive proofs?
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- What counts as a solution? Non-constructive proofs?
Hel Braun’s student-eye view.

Number theory at Frankfurt University 1933. Student of Carl Ludwig Siegel. Habilitated Göttingen 1940.
Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.
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“Perhaps I exaggerate but this is the impression I have when I compare the lectures of that time to later ones.”
Or again:
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“Still in my student days university mathematics rested strongly on mathematical talent.”
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▶ “Still in my student days university mathematics rested strongly on mathematical talent.

▶ Logic and notation were not so well established.
Or again:

▶ “Still in my student days university mathematics rested strongly on mathematical talent.

▶ Logic and notation were not so well established.

“The days are gone when one affectionately described one’s professor with ‘He said A, wrote B, meant C, and D is correct’...”
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- His lectures entirely avoided fundamental conceptual definitions, even such as limit.
- His lectures rested on lively expression and the power gained from his own studies, rather than on logic and rigor (Systematik und Strenge).
Does Emmy Noether use limits in “formale Variationsrechnung,” in her famous paper on conservation theorems? Not obvious.
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Not obvious.
The extreme difficulty of reading her dissertation.
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Computational algebraists Rebecca and Luis Garcia, Sam Houston State University.
The eulogy of Gordan

- “He compiled volumes of formulas, very well ordered but providing a minimum of text.
- His mathematical friends undertook to prepare the text for press. . . .
- They could not always produce a fully correct conception.”

“Only a few of his publications, and especially the earliest, express Gordan’s specific style: bare, brief, direct, uninterrupted theorems one after the other.”
Noether was comfortable with this.
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She would call a claim well known if someone she knew, knew it.
Noether 1916 uses a “well known” fact on polarized symmetric polynomials.
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One footnote refers to the other.
Life in Göttingen.
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No need to linger on WW I.
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Prosoviet socialism.
Weyl has her “period of relative dependence” extend to 1919 – first on Gordan, then on Hilbert.
This is fair in the sense that she let Gordan, Fischer, and Klein-Hilbert set her problems.
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But not her methods.
Her “method” circa 1915–1918:
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And Noether never claimed independence: “It is all already in Dedekind.”

What she saw in Dedekind – as she saw conservation laws in Lie.
Twentieth Century Mathematics according to Noether
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Build a community around Dedekind’s achievements.
What does mathematics deal with?
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- Dedekind: integers, real and complex numbers, sets.
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- Dedekind: integers, real and complex numbers, sets.
- Noether defers to her friend Zermelo.

What questions may legitimately be asked about such entities?

Structural relations.

What counts as a solution?

Homomorphism and isomorphism theorems.

Those are not the only solutions allowed in principle, of course. They are the preferred means of solution for Dedekind and Noether.
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The growth of mathematics, and the rise of more uniform standards, both required and permitted profusion of textbooks.
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A symptom of Noether’s closeness to this: It is easy today to get a rather close replica of her MA articles in \LaTeX, as it has fonts closely based on Springer-Verlag of the time.
It is true that there were already excellent monographs at the time and, in fact, the Bourbaki treatise was modeled in the beginning on the excellent algebra treatise of Van der Waerden. I have no wish to detract from his merit, but as you know, he himself says in his preface that really his treatise had several authors, including E. Noether and E. Artin, so that it was a bit of an early Bourbaki.
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so that it was a bit of an early Bourbaki.”
Norbert Schappacher objects strongly that all my sources are unreliable and have their own agendas. He and I agree the same is true of Weyl. We both like Alexandroff on Noether, but you can hardly avoid saying the same of him.
However:

▶ What I quote from Braun is clearly true. She just puts it well.

▶ Indeed Gordan is an extreme case, but so was Poincaré, and I could as well cite Lie or Klein as examples that I know. I doubt many less known professors were very much more like Weierstrass than these were.

▶ Norbert tells me Cordula Tollmien has found that Max Noether resented Gordan for making him do a lot of work in Gordan’s name.

▶ Far from disqualifying him as a source, this makes it more likely than I had thought before, that Max was one of those who wrote proofs for Gordan without understanding them.
What personal agenda would lead Taussky-Todd to say Noether would not recommend women for jobs?

Most likely, that Noether said exactly this to her, in declining to recommend her.

No one says Noether \textit{did} attend the women’s group meeting at the ICM 1932. Noether talked at that ICM. Taussky-odd says she told Noether about the meeting and Noether endorsed it in principle without attending.

Why would Taussky-Todd make up that Noether said women should not try to work as hard as men?

Can we believe it was resentment over Noether’s methods in number theory? Or should we rather believe Taussky-Todd resented Noether saying things just like this?
Saunders Mac Lane did not call himself a friend, but a student. He says he did not understand her lectures. Should we dismiss this as a late reconstructed memory? Or should we rather say he – whose later work would in fact utterly reorganize the material that he heard her lecture on – had a more demanding sense of “understanding” the material than most people would?