The Two Careers of Emmy Noether

A notable career in Nineteenth Century Erlangen.

1905

1916

The *long Nineteenth Century*.

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Mildred Sanderson, 1889–1914.

- Master's and Ph.D. with L.E. Dickson, Chicago.
- Thesis "Formal Modular Invariants with Applications to Binary Modular Covariants."

Noether:

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- Circolo Matematico di Palermo 1908.
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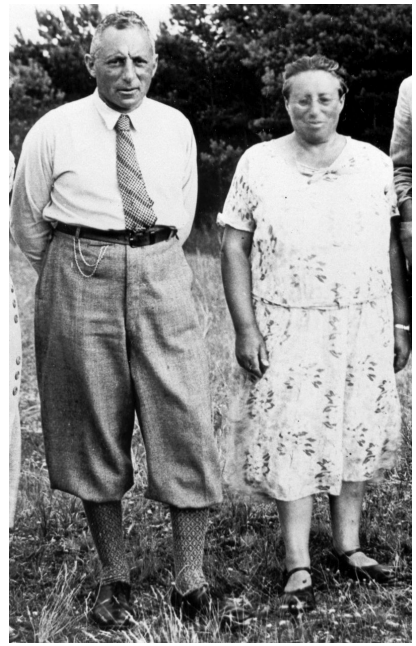
Then a different career: different place, different life, and a different century in world history and in mathematics.

Life in Erlangen

Weyl: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."



Weyl probably heard this from her brother Fritz, and it is probably true.

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Her lowest grade was "satisfactory," for practical classroom conduct.

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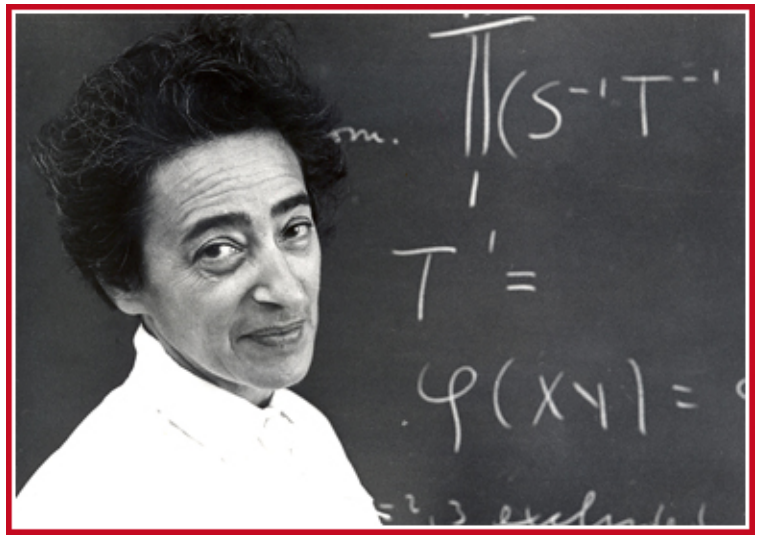

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- "She said women should not try to work as hard as men.
- She remarked that she, on the whole, only helped young men to obtain positions so they could marry and start families.
- She somehow imagined that all women were supported."

The Turn of the Century in Mathematics.

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Not (necessarily) formal foundations.

A broad consensus as to:

- What mathematics deals with: numbers?

A broad consensus as to:

- What mathematics deals with: numbers? symbols?

A broad consensus as to:

- What mathematics deals with: numbers? symbols? quantities?

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- What do we assume about them at base?
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- What counts as a solution? Non-constructive proofs?

Hel Braun's student-eye view.

Number theory at Frankfurt University 1933. Student of Carl Ludwig Siegel. Habilitated Göttingen 1940.

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"Perhaps I exaggerate but this is the impression I have when I compare the lectures of that time to later ones."

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"The days are gone when one affectionately described one's professor with 'He said A, wrote B, meant C, and D is correct'..."

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- His lectures entirely avoided fundamental conceptual definitions, even such as *limit*.
- His lectures rested on lively expression and the power gained from his own studies, rather than on logic and rigor (Systematik und Strenge).

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Not obvious.

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Computational algebraists Rebecca and Luis Garcia, Sam Houston State University.

The eulogy of Gordan

- "He compiled volumes of formulas, very well ordered but providing a minimum of text.
- His mathematical friends undertook to prepare the text for press. . . .
- They could not always produce a fully correct conception."

"Only a few of his publications, and especially the earliest, express Gordan's specific style: bare, brief, direct, uninterrupted theorems one after the other."

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She would call a claim well known if someone she knew, knew it.

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One footnote refers to the other.

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Pro-soviet socialism.

Weyl has her "period of relative dependence" extend to 1919 – first on Gordan, then on Hilbert.

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But not her methods.

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And Noether never claimed *independence*: "It is all already in Dedekind."

What *she* saw in Dedekind – as *she* saw conservation laws in Lie.

Twentieth Century Mathematics according to Noether

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Build a community around Dedekind's achievements.

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- What questions may legitimately be asked about such entities? Structural relations.
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Those are not the only solutons allowed in principle, of course. They are the preferred means of solution for Dedekind and Noether.

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A symptom of Noether's closeness to this: It is easy today to get a rather close replica of her MA articles in LaTeX, as it has fonts closely based on Springer-Verlag of the time.

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so that it was a bit of an early Bourbaki."

Norbert Schappacher objects strongly that all my sources are unreliable and have their own agendas. He and I agree the same is true of Weyl. We both like Alexandroff on Noether, but you can hardly avoid saying the same of him.

However:

- What I quote from Braun is clearly true. She just puts it well.
- Indeed Gordan is an extreme case, but so was Poincaré, and I could as well cite Lie or Klein as examples that I know. I doubt many less known professors were very much more like Weierstrass than these were.
- Norbert tells me Cordula Tollmien has found that Max Noether resented Gordan for making him do a lot of work in Gordan's name.
- Far from disqualifying him as a source, this makes it *more* likely than I had thought before, that Max was one of those who wrote proofs for Gordan without understanding them.

- What personal agenda would lead Taussky-Todd to say Noether would not recommend women for jobs?
- Most likely, that Noether said exactly this to her, in declining to recommend her.
- No one says Noether *did* attend the women's group meeting at the ICM 1932. Noether talked at that ICM. Taussky-odd says she told Noether about the meeting and Noether endorsed it in principle without attending.
- Why would Taussky-Todd make up that Noether said women should not try to work as hard as men?
- Can we believe it was resentment over Noether's methods in number theory? Or should we rather believe Taussky-Todd resented Noether saying things just like this?

Saunders Mac Lane did not call himself a friend, but a student. He says he did not understand her lectures. Should we dismiss this as a late reconstructed memory? Or should we rather say he – whose later work would in fact utterly reorganize the material that he heard her lecture on – had a more demanding sense of "understanding" the material than most people would?