Titles & Abstracts

Rolf Andreasson (Chalmers University)

Sharp bounds on the height of arithmetic Fano varieties

In the setting of Arakelov geometry, one can define a notion of height of a polarized arithmetic variety endowed with an hermitian metric. For a metrized arithmetic Fano variety, the anticanonical polarization thus gives us a natural notion of height. The height in this case is an arithmetic analogue of the degree of a Fano variety over C. Among K-semistable Fano manifolds, K. Fujita proved that the degree is maximal for projective space. Inspired by his result, we conjecture that, (after an appropriate normalization) the height of a metrized arithmetic Fano variety whose complexification is K-semistable is maximal for projective space over the integers, endowed with the Fubini-Study metric. I will present some results in favour of the conjecture for diagonal hypersurfaces, and in the toric case, highlighting connections to Kähler-Einstein metrics and K-stability along the way. This is based on joint work with Robert Berman.

Gergely Bérczi (Aarhus University)

Enumerative geometry through moduli of jets

Moduli space of jet bundles and its intersection theory plays a central role in various enumerative and complex geometry problems, including curve counting and hyperbolicity questions. In this survey I will present recent developments in various aspects of this story.

Robert Berman (Chalmers University)

Canonical heights, periods and the Hurwitz zeta function

Consider a homogeneous polynomial P with integer coefficents. Its “naive” height is, in classical Diophantine geometry, is defined as the maximum of the absolute values of the coefficients of P. More generally, in the framework of Arakelov geometry, the height is a real number, depending on the choice of a Hermitian metric on the hyperplane line bundle over the complex projective variety X, cut out by P. In a recent joint work with Rolf Andreasson we introduce a canonical height, obtained by taking the metric in question to be Kähler-Einstein - if such a metric exists. This canonical height has several useful properties. In particular, it can be expressed as a limit of periods on the N-fold products X^N, as N tends to infinity. In this talk, I will explain how this leads to an explicit expression for the canonical height of any diagonal homogeneous polynomial P in three variables. The formula involves the Hurwitz zeta function and its derivative at s=-1. Some connections to Parshin's conjectural arithmetic Miyaoka–Yau type inequality and Shimura curves will also be pointed out. No prior knowledge of Arakelov geometry will be assumed.

Harold Blum (University of Utah)

Moduli of boundary polarized Calabi-Yau pairs

While the theories of KSBA stability and K-stability have been successful in constructing compact moduli spaces of canonically polarized varieties and Fano varieties, respectively, the case of K-trivial varieties remains less well understood. I will discuss a new approach to this problem in the case of CY pairs (X,D), where D is ample, in which we

consider all semi-log-canonical degenerations. One challenge of this approach is that the set of degenerations is unbounded. Nevertheless, in some low dimensional cases, we construct a projective moduli space. This is based on work with K. Ascher, D. Bejler, K. DeVleming, G. Inchiostro, Y. Liu, and X. Wang as well as upcoming work with Y. Liu.

Xuemiao Chen (University of Waterloo)

On Vafa-Witten equations over Kaehler manifolds

I will talk about some analytic properties of solutions to the Vafa-Witten equations over compact Kaehler manifolds. Simple obstructions to the existence of nontrivial solutions are identified. The gauge theoretical compactness for the C* invariant locus of the moduli space behaves similarly as the Hermitian-Yang-Mills connections. More generally, this holds for solutions with uniformly bounded spectral covers such as nilpotent solutions. When spectral covers are unbounded, we manage to take limits of the renormalized Higgs fields which are intrinsically characterized by the convergence of the associated spectral covers. This gives a simpler proof for Taubes' results on rank two solutions over Kaehler surfaces together with a new complex geometric interpretation.

Huayi Chen (Westlake University & Paris Cité)

Arithmetic surface over a trivially valued field

In this talk, I will explain how the Arakelov geometry permits to consider a projective curve over a field as an arithmetic surface if one equips the base field with the trivial valuation. Several classical results, such Hilbert-Samuel formula and Hodge index theorem, have natural analogues in this framework. It is a joint work with Atsushi Moriwaki.

Yifan Chen (UC Berkeley)

More Complete Calabi-Yau Metrics of Calabi Type

We construct more complete Calabi-Yau metrics of Calabi type. They are the higher-dimensional analogues of ALH* gravitational instantons in two dimensions. Our work builds on and generalizes the results of Tian-Yau and HSVZ, creating Calabi-Yau metrics that are only polynomially close to the model space. We also prove the uniqueness of such metrics in a given cohomology class with fixed asymptotic behavior.

Tamás Darvas (University of Maryland)

The trace operator of quasi-plurisubharmonic functions on compact Kähler manifolds

We introduce the trace operator for quasi-plurisubharmonic functions on compact Kähler manifolds, allowing us to study the singularities of such functions along submanifolds where their generic Lelong numbers vanish. Using this construction we obtain novel Ohsawa-Takegoshi extension theorems and give applications to restricted volumes of big line bundles (joint work with Mingchen Xia).

Dennis Eriksson (Chalmers University)

On some singularity invariants from analytic torsion

I will discuss holomorphic analytic torsion and how studying its behavior close to singular fibers is related to certain singularity invariants. In the particular case of Calabi-Yau manifolds, these results sometimes give information about degenerations of Calabi-Yau, and the study thereof suggests conjectures for the mentioned singularity invariants in non-Calabi-Yau settings. This is joint work with Gerard Freixas.

Yanbo Fang (Aarhus University)

Arithmetic Hilbert-Samuel formula over a non-archimedean place

A polarised variety over a field has its geometric H-S function that counts the amont of sections; it has an asymptotic expansion (as the argument goes to infinity) whose leading term is the self-intersection number of the line bundle. Once the field is valued and the polarisation metrised, there is an arithmetic/normed analogue which is expressed as a formula for secondary invariants, formally analogous to the formula of differentiability of relative volume with a singular perturbation term. Over complex numbers such a formula has been established by Abbes-Bouche in the nineties using spectral methods for d-bar operators; we investigate the n.-A. case from a n.-A. analytic point of view.

Gerard Freixas i Montplet (École Polytechnique)

On the Deligne program

In the late 80's paper "Le déterminant de la cohomologie", Deligne proposed a program aiming to categorize the codimension one part of the Grothendieck-Riemann-Roch theorem in algebraic geometry. This is meant to be a lifting of the corresponding equality of characteristic classes to a canonical isomorphism of line bundles. Deligne treated the case of families of curves, based on results of Mumford on tautological line bundles on the moduli space of curves, and the theory of the so-called Deligne pairings. The latter was extended to higher dimensions by Elkik, who also showed how to construct canonical line bundle representatives of some fiber integrals of characteristic classes. In a joint work with Dennis Eriksson, we elaborate on Elkik's work to obtain a good functorial intersection theory valued in line bundles, and in an ongoing project we plan to complete Deligne's program in arbitrary dimension. In this talk I will motivate and present the main lines of this work.

Masafumi Hattori (Kyoto University)

Moduli space of K-stable Calabi-Yau fibrations and the positivity of CM line bundles

The characterization of K-stable varieties is well-studied when K_X is ample or X is a Calabi-Yau or Fano variety. However, K-stability of Calabi-Yau fibrations (i.e., K_X is relatively trivial) is not known much in algebraic geometry. We introduce uniform adiabatic K-stability if $f\colon (X,H) \to (B,L)$ is a fibration of polarized varieties, which means that K-stability of $(X, aH+L)$ for sufficiently small $a > 0$. In this talk, I would like to explain that uniform adiabatic K-stability of a Calabi-Yau fibration over a curve is equivalent to K-stability of the base curve in some sense. Furthermore, we construct separated moduli spaces of polarized uniformly adiabatically K-stable Calabi-Yau

fibrations over curves. If time allows, I would like to talk about positivity of CM line bundles on this moduli. This talk is based on a joint work with Kenta Hashizume.

Eiji Inoue (Riken)

Perelman entropy of Kählerian spacetime

Perelman entropy has an intriguing aspect along geodesic ray in the space of Kähler metrics. I will survey recent developments on this topic, especially a new key notion called distortion and moment measure.

Yang Li (MIT)

Calabi-Yau metrics in the intermediate complex structure limit

Calabi-Yau metrics can degenerate in a 1-parameter family by varying the complex structure, and a basic invariant is the dimension of the essential skeleton, which is an integer between 0 and n. The case of zero is the context of non-collapsed degeneration of Donaldson-Sun theory, while the case of n is the context of the SYZ conjecture. We will discuss how to describe the Kähler potential at the C^0-level in the intermediate case for a large class of complete intersection examples. This is a story about two asymptotic limits in complex geometry, the limit of complex structure degeneration, and the limit of Kodaira embeddings for very large powers of the line bundle.

Tran Trung Nghiem (Université de Montpellier)

K-stable valuations and Calabi-Yau metrics on affine spherical varieties.

In the framework of Donaldson-Sun theory, an exact Calabi-Yau metric with maximal volume growth on a smooth affine variety induces a valuation called K-stable valuation in a recent paper by Sun-Zhang. This valuation in turn degenerates the variety to a K-stable cone (i.e. admitting a conical Calabi-Yau metric), which turns out to be the asymptotic cone of the variety. I will give a partial classification result of exact Calabi-Yau metrics with maximal volume growth on rank two symmetric spaces (modulo uniqueness). Namely, I will detail the K-stable valuations on these spaces, which are in correspondence with the candidates for asymptotic cones, as well as a construction of Calabi-Yau metrics asymptotic to these cones. Among these examples, there is a class of spaces not admitting any Calabi-Yau metrics asymptotic to a given K-stable cone. If time allows, I will also try to explain why the only Calabi-Yau metrics with spherical symmetry on C^3 are the standard metric and Yang Li's metric.